Arithmetic Progression

26) Which term of the AP: 120, 116, 112.... is its first negative term?

2012/2015 [2 marks)

Let nth term of the APbe zero. Then, $a_n = 0 \Rightarrow \text{a} + (\text{n} - 1)d = 0$

So, $120 + (n - 1)(-4) = 0$

Or$4n = 124$

Or$n = 31$

∴ The first negative term of the AP is $(n+1)^{th}$ term $=32nd$ term.

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27) The ratio of the 5^{th} and 3^{rd} terms of AP $2:5$ Find the ratio of the 15^{th} and 7^{th} terms.

2014/2015 (4 Marks)

13:1

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28) In an AP, 6^{th} term is half the 4^{th} term and 3^{rd} term is 15. How many terms are needed to give a sum that is equal to 66?

2012/2014/2015 (4 Marks)

$$a_6 = \frac{1}{2} a_4$$

$a + 5d = \frac{1}{2}(a + 3d) \Rightarrow \quad a + 7d = 0$ ....(1)

Also, $\quad a_3 = a + 2d = 15$ ....(2)

From (1) *and* (2),$5d = -15 \Rightarrow \quad d = -3$

Putting, $d = -3 in\ (2), we\ get$

$$a = 15 - 2 \quad (\quad) \quad 15 \quad 6 \quad 21$$

Now$66 = \frac{n}{2}[2(21) + (n - 1)(-3)]$

$$\Rightarrow \quad 132 = n(42 - 3n + 3)$$

$\Rightarrow \quad 132 = 45n - 3n^2$

$$\Rightarrow \ n2 - 15n + 44 = 0$$

$$\Rightarrow \quad (n - 11)(n - 4) = 0 \Rightarrow \quad n = 11 \quad or \quad n = 4.$$

So, terms needed are 4 *or* 11.

In this case, the sum of $5^{th}$to 11^{th} terms will be zero.

29) In a garden bed, there are 23 roseplants in the first row, 21 are in the 2nd, 19 in 3rd row and so on. There are 5 plants in the last row. How many rows are there of rose plants? Also, find the total number of rose plants in the garden.

2012/2014/2015 (4Marks)

The number of rose plants in the 1st, 2nd, 3rd ……, last rows are 23, 21, 19…..,$a = 23, d = -2$ and $a_n = 5$

$$a_n = a + (n-1)d$$

$\Rightarrow$ $$5 = 23 + (n-1)(-2)$$

$\Rightarrow$ $$n = \frac{5-23}{-2} + 1 = 9 + 1 = 10.$$

Total number of rose plants in the flower bed

$$s_n = \frac{n}{2}[2a + (n-1)d]$$

i.e. $$s_{10} = 5(46 - 18) = 140.$$

30) The sum of first six terms of an AP is 42. The ratio of 10th term to its 30th term is 1: 3. Calculate the first term and the 13th term of AP.

Let the first term be a and common difference be d. Then,

$$s_6 = \frac{6}{2}[2a + (6-1)d] = 42$$

$$\Rightarrow 6a + 15d = 42 \quad \ldots\ldots(1)$$

According to the equation,

$$\frac{a+9d}{a+29d} = \frac{1}{3}$$

$$\Rightarrow 3a + 27d = a + 29d$$

$\Rightarrow$a=d.

Putting $d = a$ in $(1), we$ get $21a = 42 \Rightarrow a = 2 = d.$

Therefore $a13 = a + 12d = 2 + 12$.

31) The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of houses preceding the house numbered x is equal to the sum of the number of houses following it. Find the value of x.

Here, we are given that

$$\{1 + 2 + 3 + \ldots + (x - 1)\} = \{(x + 1) + (x + 2) + (x + 3) + \cdots + 49\}$$

$$\Rightarrow \left(\frac{x-1}{2}\right)\{2 \times 1 + (x - 1 - 1) \times 1\} = \left(\frac{49-x}{2}\right)\{2(x + 1) + (49 - x - 1) \times 1\}$$

$$\Rightarrow \quad (\quad)(\quad) \quad (\quad)(\quad)$$

$$\Rightarrow \quad x \quad 1 \quad x \quad 49 \quad x \quad 50 \quad x$$

$$\Rightarrow \quad {}^2 - x \quad 2450 \quad 49x \quad 50x \quad {}^2$$

$$\Rightarrow \quad 2 \quad {}^2 = 2450 \Rightarrow x^2 = 1225$$

$$\Rightarrow \quad x \quad \sqrt{\quad} \quad 35$$

Thus, value of is

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32) If the sum of the first terms of an AP is and the sum of first n terms is

Let the AP be, $a, a + d, a + 2d, \ldots$

So, $\frac{m}{2}[2a + (m - 1)d] = n$

$\Rightarrow$ $2am + m(m - 1)d] = 2n \ldots.(1)$

Also, $\frac{n}{2}[2a + (n - 1)d] = m$

$\Rightarrow$ $[2an + n(n - 1)d] = 2m$ ....(2)

Subtracting (2)from (1), We have:

$$2a(m - n) + [(m^2 - n^2) - (m - n)]d = 2(n - m)$$

$\Rightarrow$ $2a + (m + n - 1)d = -2$

So, $S_{m+n} = \frac{m+n}{2}[2a + (m + n - 1)d]$

$= \frac{m+n}{2}(-2) = -(m + n).$

Thus, it shows that the sum of the first $(m + n)$ terms of the AP is – $(m + n)$

$$\Rightarrow \quad (x-1)(2+x-2)=(49-x)(2x+2+49-x-1)$$

$$\Rightarrow \quad (x-1)(x)=(49-x)(50+x)$$

$$\Rightarrow \quad x^2-x=2450+49x-50x-x^2$$

$$\Rightarrow \quad 2x^2=2450 \Rightarrow x^2=1225$$

$$\Rightarrow \quad x=\sqrt{1225}=35$$

Thus, value of x is 35.

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32) If the sum of the first m terms of an AP is n and the sum of first n terms is m, then show that the sum of its first$(m+n)$ terms is $-(m+n)$.

Let the AP be,$a, a+d, a+2d, \ldots$

So, $\frac{m}{2}[2a+(m-1)d]=n$

$\Rightarrow$ $2am+m(m-1)d]=2n \ldots.(1)$

Also, $\frac{n}{2}[2a+(n-1)d]=m$

$\Rightarrow$ $[2an+n(n-1)d]=2m$ ....(2)

Subtracting (2)from (1), We have:

$$2a(m-n)+[(m^2-n^2)-(m-n)]d=2(n-m)$$

$\Rightarrow$ $2a+(m+n-1)d=-2$

So, $S_{m+n}=\frac{m+n}{2}[2a+(m+n-1)d]$

$=\frac{m+n}{2}(-2)=-(m+n).$

Thus, it shows that the sum of the first $(m+n)$ terms of the AP is – $(m+n)$